ABOUT THE AREA OF TRIANGLE DETERMINED BY CEVIANS OF RANK (k, l, m)

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Abstract. In this article we want to do a characterization of the areas of pedal triangles of some important points from the triangle chosen from C. Kimberling's *Encyclopedia of triangle centers.* A series of these points being points of concurrence of cevians of rank (k, l, m), of the triangle. Also, we present several equalities from these points.

1. INTRODUCTION

The barycentric coordinates were introduced in 1827 by Möbius as [3]. Barycentric coordinates are triplets of numbers (t_1, t_2, t_3) corresponding to masses placed at the vertices of a reference triangle ABC. These masses then determine a point P, which is the *geometric centroid* of the three masses and is identified with coordinates (t_1, t_2, t_3). The areas of BPC, CPA and APB triangles are proportional with barycentric coordinates t_1, t_2 and t_3. Characteristics of barycentric coordinates we can find in the monographs of C. Bradley [3], C. Coandă [4], C. Coşniţă [5], C. Kimberling [7], S. Loney [8] and to the papers of O. Bottema [2], J. Scott [14], H. Tanner [15], and P. Yiu [16]. Denote by a, b, c the lenghts of the sides in the standard order, by s the semiperimeter of triangle ABC, by $\Delta[ABC]$ the area of the triangle ABC.

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An interesting property refering to barycentric coordinates is given by Coşniţă [5], in the following way:

If the vertices P_i of a triangle $P_1P_2P_3$ have the barycentric coordinates (x_i, y_i, z_i) in relation with a triangle ABC, then the area of the triangle $P_1P_2P_3$ is

$$\Delta\left[P_1P_2P_3\right] = \Delta\left[ABC\right] \cdot \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} : \prod_{i=1}^{3}(x_i + y_i + z_i). \tag{1}$$

Also, Bottema [2], Coandă [4], Muggeridge [11] and Yiu [16] refer to the relation between the areas of the triangle $P_1P_2P_3$ and ABC, written by barycentric coordinates normalized (i.e. $x_i + y_i + z_i = 1$, for all $i = \overline{1,3}$).

Let P be a point inside of the triangle ABC. The cevian triangle DEF is defined as the triangle composed of the endpoints of the cevians though the cevian point P. If the point P has barycentric coordinates $t_1 : t_2 : t_3$, then the cevian triangle DEF has barycentric coordinates for the vertices given thus: $D(0 : t_2 : t_3), E(t_1 : 0 : t_3)$ and $F(t_1 : t_2 : 0)$. Therefore, relation (1) becomes

$$\Delta[DEF] = \frac{2t_1t_2t_3}{(t_1+t_2)(t_2+t_3)(t_3+t_1)}\Delta[ABC]. \tag{2}$$

In [9], we present the cevians of rank (k,l,m) given in following way: *If on side (BC) of a ABC unisosceles triangle a point D is taken, so that:*

$$\frac{BD}{DC} = \left(\frac{c}{b}\right)^k \cdot \left(\frac{s-c}{s-b}\right)^l \cdot \left(\frac{a+b}{a+c}\right)^m \tag{3}$$

$k, l, m \in \mathbb{R}$, then AD is called cevian of rank (k,l,m), and if $D \in BC\backslash\left[BC\right]$, so that $\frac{BD}{DC} = \left(\frac{c}{b}\right)^k \cdot \left(\frac{s-c}{s-b}\right)^l \cdot \left(\frac{a+b}{a+c}\right)^m$, $k, l, m \in \mathbb{R}^*$, then AD is called excevian of rank (k,l,m) or exterior cevian of rank (k,l,m). If ABC triangle is isosceles $(AB = AC)$, then, through convention, the cevian of rank (k,l,m) is the median from A.

In [9], shows that in a triangle the cevians of rank (k,l,m) are concurrent in the point $I(k,l,m)$ and the barycentric coordinates of $I(k,l,m)$ are:

$$a^k(s-a)^l(b+c)^m : b^k(s-b)^l(a+c) : c^k(s-c)^l(a+b)^m. \tag{4}$$

A series of points from *Encyclopedia of triangle centers* of C. Kimberling are the points of the intersection of the cevian of rank (k,l,m).

2. THE AREA OF TRIANGLE DETERMINED BY CEVIANS OF RANK (k, l, m)

Let DEF be the cevian triangle coresponding to the point $I(k, l, m)$ in relation with the triangle ABC.

Theorem 1. *There is the following relation:*
(5)
$$\Delta[DEF] = \frac{2(abc)^k[(s-a)(s-b)(s-c)]^l[(a+b)(b+c)(c+a)]^m}{\prod\limits_{cyclic}[b^k(s-b)^l(a+c)^m + c^k(s-c)^l(a+b)^m]} \cdot \Delta[ABC].$$

Proof. Taking into acount that barycentric coordinates of $I(k, l, m)$ are

$$t_1 = a^k(s-a)^l(b+c)^m : t_2 = b^k(s-b)^l(a+c)^m : t_3 = c^k(s-c)^l(a+b)^m,$$

by replacing in relation (2), we deduce the relation of the statement. □

Remark 1. *In* [9] *the notion of cevian of rank* (k, l, m) *was extended to the cevian of rank* $(k_u, k_{u+1}, ..., k_w)$ *thus:*

$$\frac{BD}{DC} = \prod_{i=u}^{w}\left(\frac{is-c}{is-b}\right)^{k_i},$$

where $u \le w, u, w \in \mathbb{Z}, k_i \in \mathbb{R}$, *for all* $i \in \{u, ..., w\}$.

Therefore, the relation (5) becomes

$$\Delta[DEF] = \frac{\prod\limits_{i=u}^{w}[(is-a)(is-b)(is-c)]^{k_i}}{\prod\limits_{cyclic}\left[\prod\limits_{i=u}^{w}(is-b)^{k_i} + \prod\limits_{i=u}^{w}(is-c)^{k_i}\right]} \cdot \Delta[ABC],$$

where the triangle DEF is the cevian triangle coresponding to the point $I(k_u, k_{u+1}, ..., k_w)$, which is the point of the intersection of cevians of rank $(k_u, k_{u+1}, ..., k_w)$.

Theorem 2. *Let* ABC *be a triangle. Denote by* D, E *and* F *respectively, the point of intersection of the cevians of rank* (k, l, m) *from* A, B, C *with the opposite sides. Let* P *be the point of intersection of the cevians of rank* (k, l, m), *and* X, Y *and* Z, *respectively, the perpendicular feet of* P *on the side* BC, CA *and* AB. *There are the following*

relations:
(6)
$$\frac{x}{a^{k-1}(s-a)^l(b+c)^m} = \frac{y}{b^{k-1}(s-b)^l(a+c)^m} = \frac{z}{c^{k-1}(s-c)^l(a+b)^m},$$
where $|PX| = x, |PY| = y, |PZ| = z$.

Proof. Since AD is the cevian of rank (k, l, m), implies the relation
$$\frac{BD}{DC} = \left(\frac{c}{b}\right)^k \left(\frac{s-c}{s-b}\right)^l \left(\frac{a+b}{a+c}\right)^m.$$
We have
$$\frac{\Delta\,[ABD]}{\Delta\,[ACD]} = \frac{BD}{DC} = \frac{c \cdot AD \cdot \sin BAD}{b \cdot AD \cdot \sin CAD} = \frac{c}{b} \cdot \frac{\sin BAD}{\sin CAD}.$$
Hence:
$$\frac{\sin BAD}{\sin CAD} = \left(\frac{c}{b}\right)^{k-1} \left(\frac{s-c}{s-b}\right)^l \left(\frac{a+b}{a+c}\right)^m$$
In the right triangles APY and APZ (see Figure 1), we have $y = AP \cdot \sin PAE = AP \cdot \sin CAD$ and $z = AP \cdot \sin FAP = AP \cdot \sin BAD$.

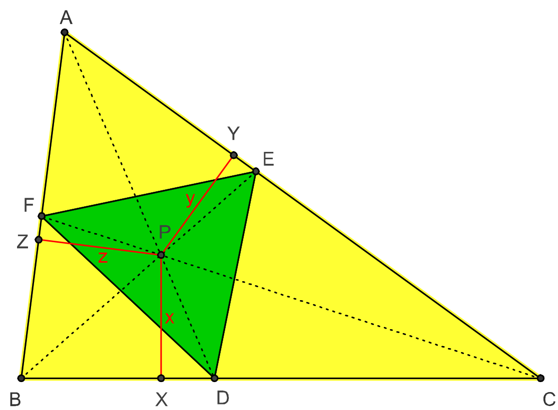

Figure 1

Thus $\frac{\sin BAD}{\sin CAD} = \frac{z}{y}$, and, therefore,
$$\frac{y}{b^{k-1}(s-b)^l(a+c)^m} = \frac{z}{c^{k-1}(s-c)^l(a+b)^m}.$$
Similary:
$$\frac{x}{a^{k-1}(s-a)^l(b+c)^m} = \frac{y}{b^{k-1}(s-b)^l(a+c)^m}$$
and the conclusion follows. □

Remark 2. *From (6), we get:*

$$\frac{ax}{a^k(s-a)^l(b+c)^m} = \frac{by}{b^k(s-b)^l(a+c)^m} = \frac{cz}{c^k(s-c)^l(a+b)^m} =$$

$$\frac{\sum ax}{\sum a^k(s-a)^l(b+c)^m} = \frac{2\Delta[ABC]}{\sum a^k(s-a)^l(b+c)^m}.$$

In [9], shows that if DEF is the cevian triangle coresponding to the point $I(k, l, m)$ in relation with the triangle ABC, Q is a point on the side EF, and X', Y' and Z' respectively, the perpendicular feet of Q on the side BC, CA and AB then we have
(7)

$$\frac{\alpha}{a^{k-1}(s-a)^l(b+c)^m} = \frac{\beta}{b^{k-1}(s-b)^l(a+c)^m} + \frac{\gamma}{c^{k-1}(s-c)^l(a+b)^m},$$

where $|QX'| = \alpha, |QY'| = \beta, |QZ'| = \gamma$. Combining (6) and (7), we obtain

$$\frac{\alpha}{x} = \frac{\beta}{y} + \frac{\gamma}{z}.$$

3. CHARACTERIZATION OF THE AREAS OF CEVIAN TRIANGLES OF SOME IMPORTANT POINTS

C. Kimberling, in [7], presents a set of points, which are written as $X(q)$. If we take $P \equiv X(q)$, where the point $X(q)$ is a point of type $I(k, l, m)$, then we obtain a series of equalities for several particular cases in relation (5). Denote by Δ the area of the triangle ABC, and by Δ' the area of the triangle DEF.

$X(q)$	$I(k,l,m)$	**Point description**	$P \equiv X(q)$ **in relation**
$X(1)$	$I(1,0,0)$	incenter	$\Delta'=\frac{2abc}{\prod(b+c)}\cdot\Delta$
$X(2)$	$I(0,0,0)$	centroid	$\Delta'=\frac{1}{4}\cdot\Delta$
$X(6)$	$I(2,0,0)$	Lemoine point	$\Delta'=\frac{2(abc)^2}{\prod(b^2+c^2)}\cdot\Delta$
$X(7)$	$I(0,-1,0)$	Gergonne point	$\Delta'=\frac{2}{sabc}\cdot\Delta^3$
$X(8)$	$I(0,1,0)$	Nagel point	$\Delta'=\frac{2}{sabc}\cdot\Delta^3$
$X(9)$	$I(1,1,0)$	mittenpunkt	$\Delta'=\frac{2abc}{s\prod[b(s-b)+c(s-c)]}\cdot$
$X(10)$	$I(0,0,1)$	Spieker point	$\Delta'=\frac{2\prod(b+c)}{\prod(2s+a)}\cdot\Delta$
$X(31)$	$I(3,0,0)$	2nd power point	$\Delta'=\frac{2(abc)^3}{\prod(b^3+c^3)}\cdot\Delta$
$X(32)$	$I(4,0,0)$	2rd power point	$\Delta'=\frac{2(abc)^4}{\prod(b^4+c^4)}\cdot\Delta$
$X(76)$	$I(-2,0,0)$	3rd Brocard point	$\Delta'=\frac{2(abc)^2}{\prod(b^2+c^2)}\cdot\Delta$
$X(86)$	$I(0,0,-1)$	Cevapoint of incenter and centroid	$\Delta'=\frac{2\prod(b+c)}{\prod(2s+a)}\cdot\Delta$
$X(321)$	$I(-1,0,1)$	isotomic conjugate of $X(81)$	$\Delta'=\frac{2\prod(b+c)}{abc\prod(\frac{a+c}{b}+\frac{a+b}{c})}\cdot\Delta$
$X(346)$	$I(0,2,0)$	isotomic conjugate of $X(279)$	$\Delta'=\frac{2}{sabc}\cdot\Delta^2$
$X(365)$	$I(\frac{3}{2},0,0)$	square root point	$\Delta'=\frac{2(abc)^{3/2}}{\prod(b^{3/2}+c^{3/2})}\cdot\Delta$
$X(366)$	$I(\frac{1}{2},0,0)$	isogonal conjugate of $X(365)$	$\Delta'=\frac{2\sqrt{abc}}{\prod(\sqrt{b}+\sqrt{c})}\cdot\Delta$
$X(560)$	$I(5,0,0)$	4th power point	$\Delta'=\frac{2(abc)^5}{\prod(b^5+c^5)}\cdot\Delta$
$X(561)$	$I(-3,0,0)$	isogonal conjugate of 4th power point	$\Delta'=\frac{2(abc)^3}{\prod(b^3+c^3)}\cdot\Delta$
$X(593)$	$I(2,0,-2)$	1st Hatzipolakis-Yiu point	$\Delta'=\frac{2(abc)^2\prod(b+c)^2}{\prod[b^2(a+b)^2+c^2(a+c)^2]}$

Remark 3. *We can see that the areas of the cevian triangles coresponding to the points* $X(6)$ *and* $X(76)$, $X(7)$ *and* $X(8)$, $X(10)$ *and* $X(86)$, $X(31)$ *and* $X(561)$, *respectively, are equals.*

4. THE CONDITION THAT THE POINT $I(k, l, m)$ BELONGS TO A LINE

Theorem 3. (***Oprea*** [1], [12], [13]) *Let D be on the side BC and l is a line not through any vertex of a triangle ABC such that l meets AB in M, AC in N, and AD in P. The following relation holds*

$$\frac{MB}{MA} \cdot \frac{DC}{BC} + \frac{NC}{NA} \cdot \frac{BD}{BC} = \frac{PD}{PA}. \tag{8}$$

Starting from the idea of a problem [13], we obtain the following:

Theorem 4. *Let ABC be a triangle. Denote by D, E and F respectively, the point of intersection of the cevians of rank (k, l, m) from A, B, C with the opposite sides. Let P be the point of concurrence of the lines AD and BE. If M and N are the point situated on the sides AB and AC, respectively, then the point P is situated on the line MN if and only if the following relation is true:*

$$\frac{MB}{MA} \cdot b^k (s-b)^l (a+c)^m + \frac{NC}{NA} \cdot c^k (s-c)^l (a+b)^m = a^k (s-a)^l (b+c)^m. \tag{9}$$

Proof. We consider the point P is on the line MN. By Van Aubel's relation in the triangle ABC (see Figure 2), we have

$$\frac{AE}{EC} + \frac{AF}{FB} = \frac{AP}{PD}. \tag{10}$$

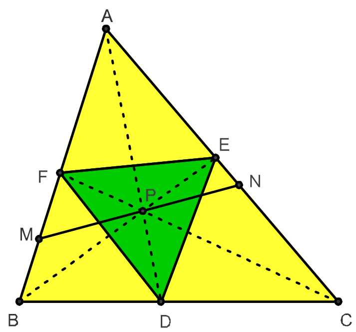

Figure 2

Since BE and CF are the cevians of rank (k, l, m), implies the relations

$$\frac{AF}{FB} = \left(\frac{b}{a}\right)^k \left(\frac{s-b}{s-a}\right)^l \left(\frac{c+a}{c+b}\right)^m \tag{11}$$

and

$$\frac{AE}{EC} = \left(\frac{c}{a}\right)^k \left(\frac{s-c}{s-a}\right)^l \left(\frac{b+a}{b+c}\right)^m. \tag{12}$$

From the relations (10), (11) and (12) we get

$$\frac{PD}{PA} = \frac{a^k(s-a)^l(b+c)^m}{b^k(s-b)^l(a+c)^m + c^k(s-c)^l(a+b)^m}. \tag{13}$$

Since AD is the cevian of rank (k, l, m), implies the relation

$$\frac{BD}{DC} = \left(\frac{c}{b}\right)^k \left(\frac{s-c}{s-b}\right)^l \left(\frac{a+b}{a+c}\right)^m,$$

so

$$\frac{BD}{BC} = \frac{c^k(s-c)^l(a+b)^m}{b^k(s-b)^l(a+c)^m + c^k(s-c)^l(a+b)^m}, \tag{14}$$

and

$$\frac{DC}{BC} = \frac{b^k(s-b)^l(a+c)^m}{b^k(s-b)^l(a+c)^m + c^k(s-c)^l(a+b)^m}. \tag{15}$$

From (8), (13), (14) and (15) we obtain (9). Conversely, we suppose that the line MN intersect the line AD in the point P'. Applying Theorem 4 to triangle ABC with cevian AD and the line MN, we have

$$\frac{MB}{MA} \cdot \frac{DC}{BC} + \frac{NC}{NA} \cdot \frac{BD}{BC} = \frac{P'D}{P'A}. \tag{16}$$

By (9) we get

$$\frac{MB}{MA} \cdot \left(\frac{b}{a}\right)^k \left(\frac{s-b}{s-a}\right)^l \left(\frac{c+a}{c+b}\right)^m + \frac{NC}{NA} \cdot \left(\frac{c}{a}\right)^k \left(\frac{s-c}{s-a}\right)^l \left(\frac{b+a}{b+c}\right)^m = 1,$$

or

$$\frac{MB}{MA} \cdot \frac{AF}{FB} + \frac{NC}{NA} \cdot \frac{AE}{EC} = 1. \tag{17}$$

Considering the triangle ADC and the transversal BE, we have by Menelaus's theorem:

$$\frac{AE}{EC} = \frac{AP}{PD} \cdot \frac{BD}{BC}. \tag{18}$$

Similarly:

$$\frac{AF}{FB} = \frac{AP}{PD} \cdot \frac{CD}{BC}. \tag{19}$$

From (17), (18) and (19) it follows that

$$\frac{MB}{MA} \cdot \frac{DC}{BC} + \frac{NC}{NA} \cdot \frac{BD}{BC} = \frac{P'D}{P'A}.$$

Comparison with (16) gives

$$\frac{PD}{PA} = \frac{P'D}{P'A}.$$

Hence the points P and P' coincide. □

If we take $P \equiv X(q)$, where the point $X(q)$ is a point of type $I(k, l, m)$, then we obtain a series of equalities for several particular cases in relation (9).

$X(q)$	$I(k, l, m)$	**Point description**	$P \equiv X(q)$ **in relation**
$X(1)$	$I(1, 0, 0)$	incenter	$b \cdot \frac{MB}{MA} + c \cdot \frac{NC}{NA} = a$
$X(2)$	$I(0, 0, 0)$	centroid	$\frac{MB}{MA} + \frac{NC}{NA} = 1$
$X(6)$	$I(2, 0, 0)$	Lemoine point	$b^2 \cdot \frac{MB}{MA} + c^2 \cdot \frac{NC}{NA} = a^2$
$X(7)$	$I(0, -1, 0)$	Gergonne point	$\frac{1}{s-b} \cdot \frac{MB}{MA} + \frac{1}{s-c} \cdot \frac{NC}{NA} = \frac{1}{s-a}$
$X(8)$	$I(0, 1, 0)$	Nagel point	$(s-b) \cdot \frac{MB}{MA} + (s-c) \cdot \frac{NC}{NA} = s$
$X(9)$	$I(1, 1, 0)$	mittenpunkt	$b(s-b) \cdot \frac{MB}{MA} + c(s-c) \cdot \frac{NC}{NA} =$
$X(10)$	$I(0, 0, 1)$	Spieker point	$(a+c) \cdot \frac{MB}{MA} + (a+b) \cdot \frac{NC}{NA} = b$
$X(21)$	$I(1, 1, -1)$	Schiffler point	$\frac{b(s-b)}{a+c} \cdot \frac{MB}{MA} + \frac{c(s-c)}{a+b} \cdot \frac{NC}{NA} = \frac{a(s-a)}{b+c}$
$X(31)$	$I(3, 0, 0)$	2nd power point	$b^3 \cdot \frac{MB}{MA} + c^3 \cdot \frac{NC}{NA} = a^3$
$X(32)$	$I(4, 0, 0)$	2rd power point	$b^4 \cdot \frac{MB}{MA} + c^4 \cdot \frac{NC}{NA} = a^4$
$X(55)$	$I(2, 1, 0)$	insimilicenter	$b^2(s-b) \cdot \frac{MB}{MA} + c^2(s-c) \cdot \frac{NC}{NA} =$
$X(56)$	$I(2, -1, 0)$	exsimilicenter	$\frac{b^2}{s-b} \cdot \frac{MB}{MA} + \frac{c^2}{s-c} \cdot \frac{NC}{NA} = \frac{a^2}{s-a}$
$X(76)$	$I(-2, 0, 0)$	3rd Brocard point	$\frac{1}{b^2} \cdot \frac{MB}{MA} + \frac{1}{c^2} \cdot \frac{NC}{NA} = \frac{1}{a^2}$
$X(86)$	$I(0, 0, -1)$	Cevapoint of $X(1)$ and $X(2)$	$\frac{1}{a+c} \cdot \frac{MB}{MA} + \frac{1}{a+b} \cdot \frac{NC}{NA} = \frac{1}{b+c}$
$X(321)$	$I(-1, 0, 1)$	isotomic conjugate of $X(81)$	$\frac{a+c}{b} \cdot \frac{MB}{MA} + \frac{a+b}{c} \cdot \frac{NC}{NA} = \frac{b+c}{a}$
$X(346)$	$I(0, 2, 0)$	isotomic conjugate of $X(279)$	$(s-b)^2 \cdot \frac{MB}{MA} + (s-c)^2 \cdot \frac{NC}{NA} =$
$X(365)$	$I(\frac{3}{2}, 0, 0)$	square root point	$b^{3/2} \cdot \frac{MB}{MA} + c^{3/2} \cdot \frac{NC}{NA} = a^{3/2}$
$X(366)$	$I(\frac{1}{2}, 0, 0)$	isogonal conjugate of $X(365)$	$\sqrt{b} \cdot \frac{MB}{MA} + \sqrt{c} \cdot \frac{NC}{NA} = \sqrt{a}$
$X(560)$	$I(5, 0, 0)$	4th power point	$b^5 \cdot \frac{MB}{MA} + c^5 \cdot \frac{NC}{NA} = a^5$
$X(3596)$	$I(-2, 1, 0)$	1st Odehnal point	$\frac{a^3}{s-a} x = \frac{b^3}{s-b} y + \frac{c^3}{s-c} z$

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